Calculus Warm Up #11-2 Evaluate the Integrals:

1) $\int \sec 3x\, dx$

2) $\int x \sin^2 x\, dx$

$\frac{1}{3}\ln|\sec 3x - \tan 3x| + C$

2) $\frac{x^2}{4} - \frac{x\sin 2x}{4} - \frac{\cos 2x}{8} + C$

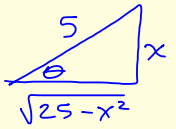
HW Questions: p. 526

1. $\int \frac{1}{(25-x^2)^{3/2}}\, dx$

$\int \frac{1}{(\sqrt{25-x^2})^3}\, dx$

$x = 5\sin\theta$

$dx = 5\cos\theta\, d\theta$



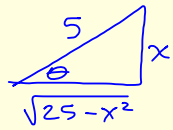
$5\cos\theta = \sqrt{25-x^2}$

2. $\int \frac{1}{x^2\sqrt{25-x^2}}\, dx = -\frac{\sqrt{25-x^2}}{25x} + C$

3. $\int \frac{\sqrt{25-x^2}}{x}\,dx$

$x = 5\sin\theta$

$dx = 5\cos\theta\,d\theta$



$5\cos\theta = \sqrt{25-x^2}$

Book answer wrong ●

$= -5\ln\left|\frac{5+\sqrt{25-x^2}}{x}\right| + \sqrt{25-x^2} + C$

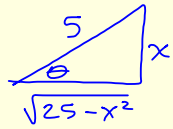
4. $\int \frac{x^2}{\sqrt{25-x^2}}\,dx$

$= \frac{25}{2}\arcsin\left(\frac{x}{5}\right) - \frac{x\sqrt{25-x^2}}{2} + C$

3. $\int \frac{\sqrt{25-x^2}}{x}\,dx$

$x = 5\sin\theta$

$dx = 5\cos\theta\,d\theta$



$5\cos\theta = \sqrt{25-x^2}$

$\int \frac{5\cos\theta}{5\sin\theta} \cdot 5\cos\theta\,d\theta$

$5\int \frac{\cos^2\theta}{\sin\theta}\,d\theta$

$5\int \frac{1-\sin^2\theta}{\sin\theta}\,d\theta$

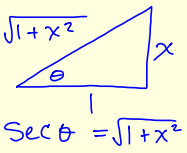
4. $\int \frac{x^2}{\sqrt{25-x^2}}\,dx$

$= \frac{25}{2}\arcsin\left(\frac{x}{5}\right) - \frac{x\sqrt{25-x^2}}{2} + C$

9. $\int x\sqrt{1+x^2}\,dx$

$x = \tan\theta$

$dx = \sec^2\theta\,d\theta$



10. $\int \frac{x^3}{\sqrt{1+x^2}}\,dx = \frac{(1+x^2)^{3/2}}{3} - \sqrt{1+x^2} + C$

$\int \frac{(\tan\theta)^3 \sec^2\theta}{\sec\theta}\,d\theta$

$\int (\tan^2\theta)\underbrace{\tan\theta\sec\theta}_{du}\,d\theta$

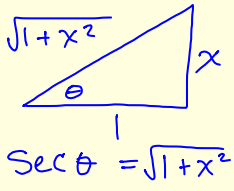
$\int (\sec^2\theta - 1)\tan\theta\sec\theta\,d\theta$

$\int \underbrace{\sec^2\theta}_{u^2}\underbrace{(\tan\theta\sec\theta\,d\theta)}_{du} - \int \tan\theta\sec\theta\,d\theta$

11. $\int \frac{1}{(1+x^2)^2}\,dx$

$x = \tan\theta$

$dx = \sec^2\theta\,d\theta$



12. $\int \frac{x^2}{(1+x^2)^2}\,dx = \frac{1}{2}\arctan x - \frac{x}{2(1+x^2)} + C$

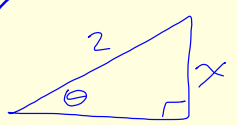
15. $\int_0^2 \sqrt{16 - 4x^2}\, dx$

$a = 2$

$u = x$

$x = 2\sin\theta$

$dx = 2\cos\theta\, d\theta$



$2\cos\theta = \sqrt{4-x^2}$

$\frac{x}{2} = \sin\theta$

$\theta = \arcsin\left(\frac{x}{2}\right)$

$= 2\int_0^2 \sqrt{4 - x^2}\, dx$

$= 2\int 2\cos\theta \cdot 2\cos\theta\, d\theta$

$= 8\int \cos^2\theta\, d\theta$

$= \frac{8}{2}\int (1 + \cos 2\theta)\, d\theta$

$= 4\left[\theta + \frac{1}{2}\int 2\cos 2\theta\, d\theta\right]$

$= 4\theta + 2\sin 2\theta$

$= \left[4\arcsin\left(\frac{x}{2}\right) + 2\cdot 2\left(\frac{x}{2}\right)\left(\frac{\sqrt{4-x^2}}{2}\right)\right]_0^2$

9.4 Day 2

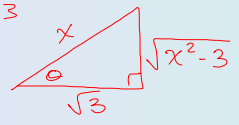
Radical Form: $\sqrt{u^2 - a^2}$

Let u = a sec θ →

$\sec\theta = \frac{u}{a}$ (hyp / adj)

Example: $u = x$ $a = \sqrt{3}$

$$\int_{\sqrt{3}}^{2} \frac{\sqrt{x^2 - 3}}{x}\, dx$$



$x = \sqrt{3}\sec\theta$

$dx = \sqrt{3}\sec\theta\tan\theta\, d\theta$

$\theta = \operatorname{arcsec}\left(\frac{x}{\sqrt{3}}\right)$

$\sqrt{3}\tan\theta = \sqrt{x^2 - 3}$

lower limit	upper limit
for $x = \sqrt{3}$	for $x = 2$
$\theta = 0$	$x = \frac{\pi}{6}$

$\int_0^{\pi/6} \frac{\sqrt{3}\tan\theta}{\sqrt{3}\sec\theta} \cdot \sqrt{3}\sec\theta\tan\theta\, d\theta$

$\sqrt{3}\int_0^{\pi/6} \tan^2\theta\, d\theta$

$\sqrt{3}\int_0^{\pi/6} (\sec^2\theta - 1)\, d\theta$

$\sqrt{3}\left[\tan\theta - \theta\right]_0^{\pi/6}$

$\sqrt{3}\left(\frac{1}{\sqrt{3}} - \frac{\pi}{6} - 0\right)$

$1 - \frac{\sqrt{3}\pi}{6}$

Completing the Square:

$$\int \frac{1}{(x^2-4x)^{3/2}}\,dx$$

$x^2-4x+4-4$

$(x-2)^2-2^2$

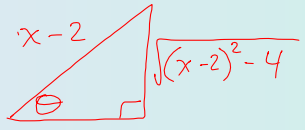
$u = x-2$

$du = dx$

$a = 2$

$$\int \frac{1}{\left(\sqrt{(x-2)^2-2^2}\right)^3}\,dx$$

$x-2 = 2\sec\theta$



$2\tan\theta = \sqrt{(\quad)^2-4}$

$x = 2\sec\theta + 2$

$dx = 2\sec\theta\tan\theta\,d\theta$

$$\int \frac{1\cdot 2\sec\theta\tan\theta\,d\theta}{(2\tan\theta)^3}$$

$$\frac{1}{4}\int \frac{\sec\theta}{\tan^2\theta}\,d\theta$$

$$\frac{1}{4}\int \frac{1}{\cos\theta}\cdot\frac{\cos^2\theta}{\sin^2\theta}\,d\theta$$

$$\frac{1}{4}\int \frac{\cos\theta}{\sin^2\theta}\,d\theta$$

$u = \sin\theta$

$du = \cos\theta\,d\theta$

Classwork: p. 526, # 5 - 8

Answers:

5. $\ln\left|x+\sqrt{x^2-4}\right| + C$

6. $\sqrt{x^2-4} - 2\operatorname{arcsec}\frac{x}{2} + C$

7.

8.

HW: p. 526,

17 - 33 odd, skip 23